Theorem: For any square matrix A with real number elements then

a. $(A + A^T)$ is symmetric

b. $(A - A^T)$ is skew-symmetric

How to calculate?

$$A = \frac{1}{2}[(A + A^T) + (A - A^T)]$$

$$\Longrightarrow A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$$

$$\Longrightarrow A = \text{Symmertic matrix} + \text{Skew Symmertic matrix}$$

Express the matrix $A = \begin{bmatrix} 9 & 2 & 3 \\ 5 & -1 & 6 \\ 4 & 0 & -8 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

Given that, $A = \begin{bmatrix} 9 & 2 & 3 \\ 5 & -1 & 6 \\ 4 & 0 & -8 \end{bmatrix}$ then

$$A^T = \begin{bmatrix} 9 & 5 & 4 \\ 2 & -1 & 0 \\ 3 & 6 & -8 \end{bmatrix}$$

$$(A + A^T) = \begin{bmatrix} 9 & 2 & 3 \\ 5 & -1 & 6 \\ 4 & 0 & -8 \end{bmatrix} + \begin{bmatrix} 9 & 5 & 4 \\ 2 & -1 & 0 \\ 3 & 6 & -8 \end{bmatrix}$$

$$\Rightarrow (A + A^T) = \begin{bmatrix} 9+9 & 2+5 & 3+4 \\ 5+2 & -1-1 & 6+0 \\ 4+3 & 0+6 & -8-8 \end{bmatrix}$$

$$\Rightarrow (A + A^T) = \begin{bmatrix} 18 & 7 & 7 \\ 7 & -2 & 6 \\ 7 & 6 & -16 \end{bmatrix}$$

So $\frac{1}{2}(A + A^T) = \begin{bmatrix} 9 & \frac{7}{2} & \frac{7}{2} \\ \frac{7}{2} & -1 & 3 \\ \frac{7}{2} & 3 & -8 \end{bmatrix}$

$$(A - A^T) = \begin{bmatrix} 9 & 2 & 3 \\ 5 & -1 & 6 \\ 4 & 0 & -8 \end{bmatrix} + \begin{bmatrix} 9 & 5 & 4 \\ 2 & -1 & 0 \\ 3 & 6 & -8 \end{bmatrix}$$

$$\Rightarrow (A - A^T) = \begin{bmatrix} 9-9 & 2-5 & 3-4 \\ 5-2 & -1+1 & 6-0 \\ 4-3 & 0-6 & -8+8 \end{bmatrix}$$

$$\Rightarrow (A - A^T) = \begin{bmatrix} 0 & -3 & -1 \\ 3 & 0 & 6 \\ 1 & -6 & 0 \end{bmatrix}$$

So $\frac{1}{2}(A - A^T) = \begin{bmatrix} 0 & -\frac{3}{2} & -\frac{1}{2} \\ \frac{3}{2} & 0 & 3 \\ \frac{1}{2} & -3 & 0 \end{bmatrix}$

Hence, $A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$

$$\Longrightarrow A = \begin{bmatrix} 9 & \frac{7}{2} & \frac{7}{2} \\ \frac{7}{2} & -1 & 3 \\ \frac{7}{2} & 3 & -8 \end{bmatrix} + \begin{bmatrix} 0 & -\frac{3}{2} & -\frac{1}{2} \\ \frac{3}{2} & 0 & 3 \\ \frac{1}{2} & -3 & 0 \end{bmatrix}$$

$$\Longrightarrow A = \text{Symmertic matrix} + \text{Skew Symmertic matrix}$$

Express the following matrices as the sum of a symmetric and a skew symmetric matrix.

$A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$	$A = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$	$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$